Math 1250G Final Exam Review

Updated Fall 2022

- ***Calculators are allowed on exam.***
- ***Exact, simplified solutions are required, unless otherwise stated.***
- ***All necessary work to arrive at a solution must be shown. If necessary steps are missing, no credit will be given for the problem.***
- ***When applicable, correct units must be given.***

1. Find the difference quotient, $\frac{f(x+h)-f(x)}{h}$ for the given functions.

   $a)\ f(x) = x^2 + 7x - 4$ $b)\ f(x) = \frac{2}{x+3}$

2. Rewrite the following equations in standard form by completing the square and state the center and the radius. Graph each circle.

   $a)\ x^2 + y^2 + 6x + 2y + 6 = 0$ $b)\ x^2 + y^2 - 6y - 7 = 0$

3. Given $x = 4$ is one solution of $x^3 - 17x + 4 = 0$, algebraically determine all remaining solutions. Leave answers in exact, simplified form.

4. Find the complete factorization of $f(x) = 3x^3 - 10x^2 - x + 12$, given $x = \frac{4}{3}$ is a root of $f(x)$.

5. For the given rational functions, determine the following:

   i. Vertical, horizontal, and slant asymptotes.

   ii. x and y intercepts.

   iii. Graph the functions, label all asymptotes, intercepts, and at least 2 points on each piece of the graph.

   $a)\ f(x) = \frac{2x^2-5x+7}{x-2}$ b) $f(x) = \frac{x+3}{x^2+2x-8}$

6. Solve the following logarithmic and exponential equations. Write exact solutions.

$a)\ e^{4x-5} - 7 = 11{,}243$ $b)\ 7^{2x+1} = 3^{x+2}$

$c)\ log_3 y + 3log_3 y^2 = 14$ $d)\ log_4(2x+1) - log_4(x-3) = log_4(x+5)$

7. The population of Africa was 807 million in 2000, and grew to 1,052 million in 2011.Use the exponential growth model $A(t) = A_0 e^{kt}$ to find the exponential growth function that models this data. In what year will Africa's population reach 2 billion (2,000 million)?

8. The function $T(t) = 13.4\ln(t) - 11.6$ models the temperature increase, *T*, in an enclosed vehicle after *t* minutes when the outside air temperature is between 72°F and 92°F.

   a) Find the temperature increase, to the nearest degree, after 15 minutes.
   b) Find how long it takes, to the nearest minute, to reach a temperature increase of 35°F.

9. Without the use of a calculator, find the values of the six trigonometric functions at each angle.

a) $\frac{\pi}{6}$ b) $\frac{7\pi}{4}$ c) $\frac{2\pi}{3}$

10. Use the given information to find the exact value of each of the remaining six trigonometric functions.

a) $cot\theta = \frac{1}{3},\ \ \pi < \theta < \frac{3\pi}{2}$ b) $sin\theta = \frac{2}{5},\ \ \frac{\pi}{2} < \theta < \pi$

11. Find θ exactly, where $0 \le \theta < 2\pi$.

$a)\ sin\theta = -\frac{1}{2}$ $b)\ tan\theta = 1$

12. Find the amplitude, period, phase shift, vertical shift, and determine if the graph has a reflection. Then graph one period, beginning at the phase shift. Scale the graph using the five key points.

$a)\ y = 2\cos(x) + 1$ $b)\ y = \sin\left(2x - \frac{\pi}{3}\right) - 2$ $c)\ y = -3\cos(x + \pi)$

13. Use a sum or difference formula to find the exact value of the following:

$a)\ \tan\left(\frac{11\pi}{12}\right)$ $b)\ \sin 105°$

14. Use a half – angle formula to find the exact value of the following.

$a)\ \cos\left(\frac{11\pi}{12}\right)$ $b)\ \tan(15°)$

15. If $sin\theta = -\frac{4}{5}$, and $\pi < \theta < \frac{3\pi}{2}$, find

a) $\sin 2\theta$ $b)\ \cos\frac{\theta}{2}$

16. Solve the following triangles. Round lengths of sides to the nearest tenth and angle measures to the nearest degree.

$a)\ B = 45°, A = 60°, a = \sqrt{6}$ $b)\ b = 11, c = 20,\ A = 37°$

17. Solve each equation on the interval $[0,2\pi)$.

$a)\ \cos(2\theta) = \frac{1}{2}$ $b)\ \cos(2\theta) + \sin(\theta) = 0$ $c)\ 2sin^2\theta + cos\theta = 1$

18. Find the polar coordinates for each rectangular point.

$a)\ (-2,-2)$ $b)\ (7,0)$ $c)\ (0,-9)$

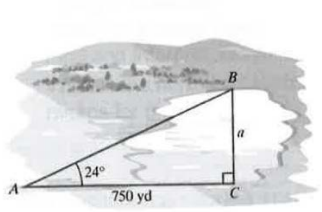
19. Find the rectangular points for each polar coordinate.

$a)\ \left(4,\frac{5\pi}{4}\right)$ $b)\ \left(\frac{1}{2},\frac{2\pi}{3}\right)$

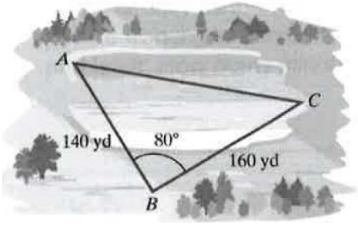
20. Convert the rectangular equations into a polar equation.

$a)\ x^2+y^2=16$ $b)\ 3x+8y=5$

21. The distance, a, across a lake is unknown. To find the distance, a surveyor took the measurements shown in the figure. What is the distance across the lake? Round to the nearest whole number.



22. Find the distance from A to C in the given picture. Round to the nearest yard.



23. Suppose you go for a hike on a bearing of S 31° W for 2.3 miles. Then you turn 90° clockwise and hike 3.5 miles on a bearing of N 59° W. At that time, what is your bearing from your start point? Round to the nearest degree.

24. Two fire lookout stations are 13 miles apart, with station B directly east of station A. Both stations spot a fire. The bearing of the fire from station A is N 35° E and the bearing of the fire from station B is N 49° W . How far to the nearest tenth of a mile, is the fire from station B?

25. Let **v** be the vector from initial point $P_1 = (-3,2)$ to terminal point $P_2 = (-5,-1)$. Write **v** in terms of **i** and **j**.

26. Let $\mathbf{u} = 3\boldsymbol{i} - 2\boldsymbol{j}$ and $\mathbf{v} = -4\boldsymbol{i} + 4\boldsymbol{j}$. Find the vector **u** + **v**.

27. Given $\mathbf{v} = 5\boldsymbol{i} + \boldsymbol{j}$ and $\mathbf{w} = \boldsymbol{i} + 5\boldsymbol{j}$, find the following.
    a. $\boldsymbol{v} \cdot \boldsymbol{w}$
    b. $\boldsymbol{v} \cdot \boldsymbol{v}$

28. Given the equation of the ellipse, determine the center, vertices, and foci. Make a graph, labeling all points found.

    a) $\frac{x^2}{9} + \frac{y^2}{49} = 1$ b) $\frac{(x-2)^2}{25} + \frac{(y+3)^2}{4} = 1$

29. Given the equation of the hyperbola, determine the center, transverse axis, vertices, asymptotes, and foci. Make a graph, labeling all points found.

    a) $\frac{x^2}{16} - \frac{y^2}{25} = 1$ b) $\frac{(y-4)^2}{16} - \frac{(x+2)^2}{9} = 1$

30. A satellite dish is in the shape of a parabolic surface. Signals coming from a satellite strike the surface of the dish and are reflected to the focus, where the receiver is located. The satellite dish has a diameter of 10 feet and a depth of 2 feet. How far from the base of the dish should the receiver be placed?

Solutions

1. a. $2x + h + 7$ b. $\dfrac{-2}{(x+3)(x+h+3)}$

2. a. $(x+3)^2 + (y+1)^2 = 4$ $Center\ (-3,-1)$ $radius = 2$

   b. $x^2 + (y-3)^2 = 16$ $Center\ (0,3)$ $radius = 4$

3. $\{-2+\sqrt{5}, -2-\sqrt{5}\}$

4. $f(x) = (3x-4)(x-3)(x+1)$

5. a. Vertical Asymptote: $x = 2$, Horizontal Asymptote: None,
   Slant Asymptote: $y = 2x - 1$
   Graph passes through: $(-3,-8), (0,-3.5), (1,-4), (3,10), (4,9.5), (7,14)$
   b. Vertical Asymptotes: $x = -4, x = 2$, Horizontal Asymptote: $y = 0$, Slant Asymptote: None
   Graph passes through: $(-8,-.125), (-6,-.1875), (-3,0), (0,-.375), (6,.225), (12, .09375)$

6. a. $x = \dfrac{ln(11{,}250) + 5}{4}$ b. $x = \dfrac{2ln(3) - ln(7)}{2ln(7) - ln(3)}$

   c. $y = 9$ d. $x = 4$

7. Equation: $A(t) = 807e^{0.0241t}$
   Africa's population will reach 2 billion in 2038.

8. a. The temperature increase after 15 minutes is 25°F.
   b. It will take 32 minutes to reach a temperature increase of 35°F.

9. a. $sin\dfrac{\pi}{6} = \dfrac{1}{2}, cos\dfrac{\pi}{6} = \dfrac{\sqrt{3}}{2}, tan\dfrac{\pi}{6} = \dfrac{\sqrt{3}}{3}, csc\dfrac{\pi}{6} = 2, sec\dfrac{\pi}{6} = \dfrac{2\sqrt{3}}{3}, cot\dfrac{\pi}{6} = \sqrt{3}$

   b. $sin\ \dfrac{7\pi}{4} = -\dfrac{\sqrt{2}}{2}, cos\ \dfrac{7\pi}{4} = \dfrac{\sqrt{2}}{2}, tan\ \dfrac{7\pi}{4} = -1, csc\ \dfrac{7\pi}{4} = -\sqrt{2}, sec\ \dfrac{7\pi}{4} = \sqrt{2}, cot\ \dfrac{7\pi}{4} = -1$

   c. $sin\dfrac{2\pi}{3} = \dfrac{\sqrt{3}}{2}, cos\dfrac{2\pi}{3} = -\dfrac{1}{2}, tan\dfrac{2\pi}{3} = -\sqrt{3}, csc\dfrac{2\pi}{3} = \dfrac{2\sqrt{3}}{3}, sec\dfrac{2\pi}{3} = -2, cot\dfrac{2\pi}{3} = \dfrac{\sqrt{3}}{3}$

10. a. $cot\theta = \frac{1}{3}, tan\theta = 3, sin\theta = \frac{-3\sqrt{10}}{10}, cos\theta = \frac{-\sqrt{10}}{10}, csc\theta = \frac{-\sqrt{10}}{3}, sec\theta = -\sqrt{10}$

b. $sin\theta = \frac{2}{5}, cos\theta = \frac{-\sqrt{21}}{5}, tan\theta = \frac{-2\sqrt{21}}{21}, csc\theta = \frac{5}{2}, sec\theta = \frac{-5\sqrt{21}}{21}, cot\theta = \frac{-\sqrt{21}}{2}$

11. a. $\left\{\frac{7\pi}{6}, \frac{11\pi}{6}\right\}$ b. $\left\{\frac{\pi}{4}, \frac{5\pi}{4}\right\}$

12. a. Amplitude: 2
Period: 2π
Phase Shift: None
Vertical Shift: Up 1
Reflection: None
Key Points: $(0,3), \left(\frac{\pi}{2}, 1\right), (\pi, -1), \left(\frac{3\pi}{2}, 1\right) (2\pi, 3)$

b. Amplitude: 1
Period: π
Phase Shift: $\frac{\pi}{6}$
Vertical Shift: Down 2
Reflection: None
Key Points: $\left(\frac{\pi}{6}, -2\right), \left(\frac{5\pi}{12}, -1\right), \left(\frac{2\pi}{3}, -2\right), \left(\frac{11\pi}{12}, -3\right) (\frac{7\pi}{6}, -2)$

c. Amplitude: 3
Period: 2π
Phase Shift: $-\pi$
Vertical Shift: None
Reflection: Yes
Key Points: $(-\pi, -3), \left(-\frac{\pi}{2}, 0\right), (0,3), \left(\frac{\pi}{2}, 0\right) (\pi, -3)$

13. a. $\tan\left(\frac{11\pi}{12}\right) = \frac{1-\sqrt{3}}{1+\sqrt{3}} = -2+\sqrt{3}$ b. $\sin 105° = \frac{\sqrt{6}+\sqrt{2}}{4}$

14. a. $\cos\left(\frac{11\pi}{12}\right) = -\frac{\sqrt{2+\sqrt{3}}}{2}$ b. $\tan(15°) = 2-\sqrt{3}$

15. a. $\sin 2\theta = \frac{24}{25}$ b. $\cos\frac{\theta}{2} = -\frac{\sqrt{5}}{5}$

16. a. $A = 60°, B = 45°, C = 75°$
$a = 2.4, b = 2, c = 2.7$
b. $a = 13.0, b = 11, c = 20$
$A = 37°, B = 31°, C = 112°$

17. a. $\left\{\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}\right\}$ b. $\left\{\frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}\right\}$ c. $\left\{0, \frac{2\pi}{3}, \frac{4\pi}{3}\right\}$

18. a. $\left(2\sqrt{2}, \frac{5\pi}{4}\right)$ b. $(7,0)$ c. $\left(9, \frac{3\pi}{2}\right)$

19. a. $\left(-2\sqrt{2}, -2\sqrt{2}\right)$ b. $\left(-\frac{1}{4}, \frac{\sqrt{3}}{4}\right)$

20. a. $r = 4$ b. $r = \frac{5}{3cos\theta + 8sin\theta}$

21. 334 $yards$

22. 193 $yards$

23. S 87.7° W

24. 10.7 miles

25. $\mathrm{v} = -2i - 3j$

26. $\mathrm{v} = -i + 2j$

27. a. $v \cdot w = 10$
b. $v \cdot v = 26$

28. a. Center: $(0,0)$, Vertices: $(3,0), (-3,0), (0,7), (0,-7)$, Foci: $\left(0,2\sqrt{10}\right), \left(0,-2\sqrt{10}\right)$
b. Center: $(2,-3)$, Vertices: $(7,-3), (-3,-3), (2,-1), (2,-5)$
Foci: $\left(2+\sqrt{21},-3\right), \left(2-\sqrt{21},-3\right)$

29. a. Center: $(0,0)$, Horizontal Transverse Axis, Vertices: $(4,0), (-4,0)$
Foci: $\left(\sqrt{41},0\right), \left(-\sqrt{41},0\right)$
b. Center: $(4,-2)$, Vertical Transverse Axis, Vertices: $(4,2), (4,-6)$
Foci: $(4,3), (4,-7)$

30. The receiver should be placed $3\frac{1}{8}$ feet from the base of the dish.